

The Nitty-Gritty of CBOT[®] DJIASM Futures Index Arbitrage

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In a market in which interest rates have reached record lows, the price of nearby CBOT[®] DJIASM futures is more likely to be at a discount relative to the DJIASM than at a premium. If the futures price goes to a premium, chances are it will reverse its course, whether in a matter of minutes or days. Thus it is now the appearance of a premium, rather than a discount, which signals profit opportunities to stock index arbitrageurs. Other than this, the basic principles of a stock index arbitrage strategy have stayed pretty much the same: when a stock index futures price is considered "rich" relative to the stock index, arbitrageurs sell futures and buy the equivalent basket of index stocks. A "cheap" futures price triggers the reverse program, long the futures and short the basket. While these rules are straightforward, implementation requires attention to institutional features. These features are the nitty-gritty details that determine whether the futures-cash price relationship is misaligned after transaction costs are taken into account.

A Short History of Stock Index Arbitrage

Stock index arbitrage programs evolved in the 1980s in tandem with trading in stock index futures. The volume of arbitrage trading has grown steadily, parallel to volume in the futures and more recently, to volume in exchange traded funds (ETFs). To illustrate, the NYSE reports that 8.4% of NYSE volume in the first week of July 2002 was related to index arbitrage programs.¹ Just four years ago, this percentage was 5%. There is also active arbitrage based on stocks traded on Nasdaq and on electronic communications networks (ECNs.)

Stock index arbitrage was originally the preserve of a concentrated group of large banks with efficient access to the NYSE and the capacity to trade the large positions required to make S&P 500 arbitrage viable. While major banks are still heavily involved, new types of firms have entered the field. These include hedge funds as well as a fringe of smaller proprietary firms. These firms tend to specialize in arbitrage of Nasdaq 100 futures ETFs and their trading is concentrated on ECNs such as Island.

The bulk of arbitrage trading is still based on the S&P 500, but the set of stock indices that can be arbitrated has expanded, particularly in the last two years following the explosion of ETFs based on all segments of the U.S. market. The launch of CBOT DJIA futures in October 1997 opened up the field for smaller arbitrage programs. Active arbitrage in CBOT DJIA contracts has been facilitated by two factors, the manageable size of the CBOT DJIA contract and the simple design of the DJIA. CBOT DJIA futures are approximately one-third the size of S&P 500 futures and the DJIA is composed of equal numbers of shares of 30 extremely liquid blue-chip stocks. These specifications scale DJIA futures stock index arbitrage programs to an accessible size and reduce tracking errors and transaction costs. The introduction of CBOT mini-sized Dow futures that are half the size of the large contract and fully electronic will further facilitate arbitrage. Now to the nitty-gritty.

¹

Fair Value of CBOT DJIA Futures

Abstracting from transaction costs and from risk related to interest rate variation or execution slippage, arbitrage of CBOT DJIA futures hinges on their "fair value". What the market calls the "fair value" of a CBOT DJIA futures contract is a futures price that rules out a profitable arbitrage between the cash DJIASM and the futures price. The possibility of arbitrage arises because it is possible to replicate a position in the cash index with a leveraged futures position. This is what links their prices. The DJIA cash index is the price of investing today in a portfolio of DJIA stocks. The price of a CBOT DJIA futures contract is the price of investing in the same portfolio at the expiration date of the futures contract. The differences between the two are:

- (1) The DJIA price is settled today while settlement of the CBOT DJIA futures price is deferred.
- (2) The futures price does not include DJIA dividends distributed before the futures expiration date.

To price these differences, the fair value of the futures price equals the DJIA plus the cost of financing the DJIA portfolio until the futures expiration date less the expiration value of dividend distributions. Formally, if a futures contract expires in T days, the relevant annualized short-term rate of interest is r, and D represents DJIA ex-dividends to expiration, in index points, the fair value of CBOT DJIA futures is:

$$F^* = \text{DJIA} (1 + rT/360) - D$$

For example, on August 28, 2002, at 2:17 p.m., Bloomberg's LIBOR-based estimate of the fair value of September 2002 futures was:

$$8644.22 = 8644.78(1 + 0.0182 \cdot 21/360) - 9.74$$

Still ignoring market frictions, arbitrageurs intervene whenever the futures price deviates from the fair value. If the futures price is above fair value, e.g. 8650 at 2:17 p.m. on August 28, arbitrageurs sell futures contracts and buy an equivalent basket of DJIA stocks. On August 28, 2002 the basket equivalent to one CBOT DJIA futures included $69 \approx 10 \cdot 6.9374 = 10/0.14418073$ shares of each of the 30 DJIA stocks, where .14418073 was the index divisor. The basket equivalent to CBOT mini-sized Dow futures included half that number of shares. The basket is financed at the prevailing rate of interest (LIBOR in the example above) and dividends are reinvested until expiration. At expiration, the futures position is closed out at the final settlement price of CBOT DJIA futures, a Special Opening Quotation of the DJIA (SOQ) based on opening prices³ of the stocks; the stocks are sold at the same prices with Market on Open orders and the cash loan at LIBOR is repaid. The profit locked in for a full-size CBOT DJIA futures contract is $\$10 \times (8650 - 8644.22) = \57.80 .

Conversely, if the futures price is below fair value, arbitrageurs buy futures contracts and short the equivalent DJIA basket. To short, one must borrow stocks from a securities broker. The stocks are sold and the cash proceeds from this sale are invested at LIBOR. Dividends that have to be paid out to the lending broker are also financed at LIBOR. At expiration, the futures position is closed out and the basket of stocks is purchased and returned to the lending broker.

Historical Mispricing Relative to Fair Value

Unrestricted, the arbitrage just described would eliminate all mispricing of the futures contract relative to fair value. In practice, arbitrage is costly to execute and never perfectly riskless. Pricing errors therefore persist. Figure 1 shows the daily mispricing of nearby CBOT[®] DJIASM futures since the launch of the contract. Mispricing was sometimes fairly high in the first two years but started to stabilize in the third quarter of 1999. It has since stayed in an interval of about 50 basis points but is much smaller on average. As Table 1 shows, year to date, the mean absolute mispricing is 14 basis points. Figure 2 shows the mispricing for September 2002 futures at one-minute intervals on a recent and particularly volatile day, July 15, 2002. The highest pricing errors for the day occurred near the open and the close, periods when volatility usually peaks.

TABLE 1			
CBOT Dow Nearby Futures			
Absolute Mispricing in Percents			
	Mean	Minimum	Maximum
1998	0.22%	0.00%	1.76%
1999	0.31%	0.00%	1.54%
2000	0.19%	0.01%	2.18%
2001	0.15%	0.00%	0.61%
2002	0.14%	0.00%	0.62%

Figure 1
Percent Mispricing of CBOT DJIA Futures
Nearby Contract, Daily Snapshot at Close
10/06/1997 - 9/04/2002

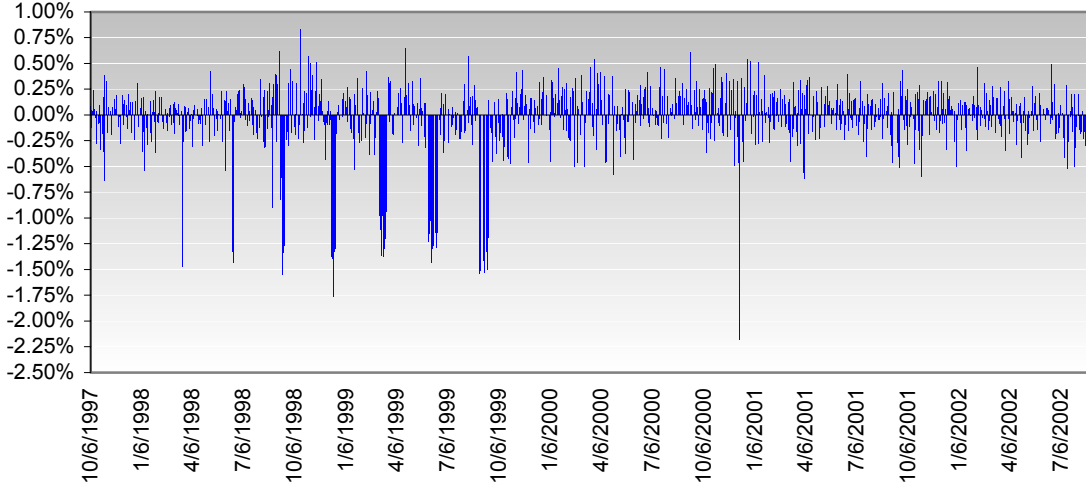
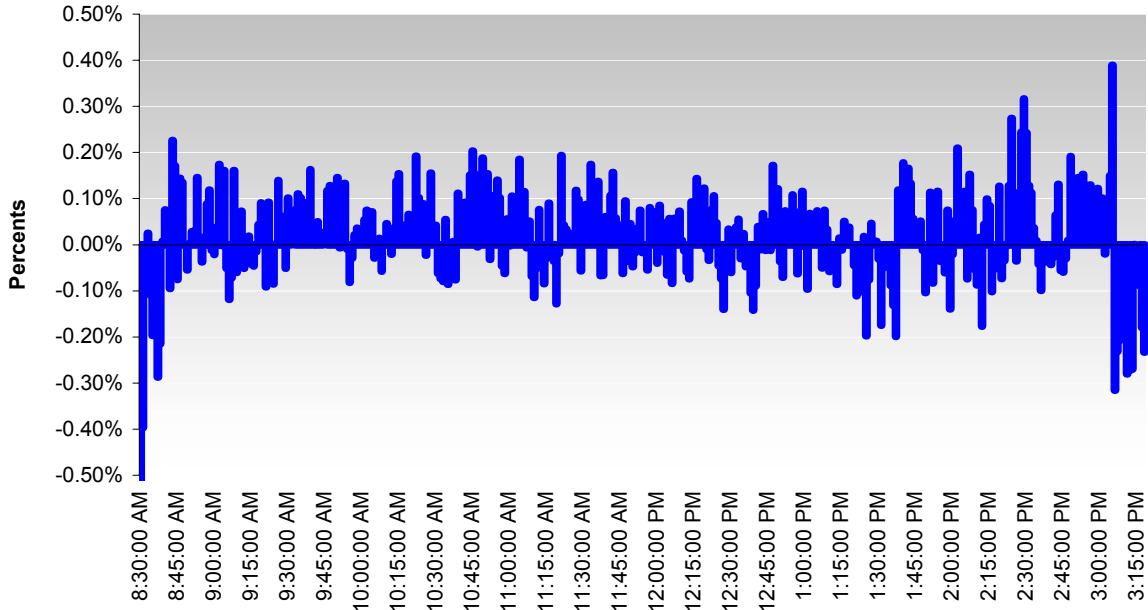


Figure 2
CBOT DJIA Nearby Futures Mispricing
July 15, 2002



Characteristics of CBOT DJIA futures mispricing are that (1) mispricing increases with volatility, (2) the futures price is more often underpriced than overpriced. This is typical of stock index futures because shorting stocks involves larger transaction costs and trading impediments than buying stocks, and (3) pricing errors are sometimes high on dividend ex-dates. A priori, it seems there should be no relation between the two. On ex-dates, the index decreases by the amount of the dividend, but this should not affect the fair value of the futures unless the ex-dividend date or dividend amount conveys new information. Several empirical studies have reported abnormal stock returns in the five to six days preceding ex-dividend dates. The probable cause is tax arbitrage by certain investor clienteles, mostly corporations with favorable dividend taxation rates. It is possible that uncertainty about the effective tax rates reflected in the price of a stock going ex-dividend affects stock index futures prices.

Executing an Arbitrage Program: Frictions and Risks

This section reviews the steps of an arbitrage program and some of the inherent frictions and risks of execution.

Size of Program: The size of an arbitrage is unlimited in theory but limited in practice by borrowing constraints. The minimum feasible arbitrage program includes 1 CBOT mini-sized Dow futures and the equivalent basket but actual arbitrage programs are often much larger, although they may be executed in smaller batches to avoid market impact.

Holding Period: To be riskless, an arbitrage position must be held until expiration. A 1990 NYSE study by George Sofianos nevertheless reported that 70% of arbitrage positions are closed out before expiration. Early liquidations occur upon a reversal of the original mispricing, either on the same day or a few days later.

Spread Ratio: A stock index arbitrage is a spread between stock index futures and an equivalent basket of index stocks. To obtain a perfectly hedged and riskless position, the spread must be perfectly balanced, i.e., the dollar exposures on the cash and futures sides have to be equal. This is usually not feasible because it is not possible to trade either fractions of futures or fractions of stocks. To illustrate, on September 4, 2002, the DJIA basket equivalent to one CBOT DJIA futures contract consisted of 69.3574 shares of DJIA stocks. To be tradeable, this number must be rounded to 69. Furthermore, since it is easier to trade shares in round-lots (multiples of 100 shares) it is not that convenient to trade the equivalent DJIA basket even after rounding. Conversely, the number of futures contracts equivalent to a round lot is 1.441807.

Tailing

A related consideration is that to obtain perfect balancing, the futures side needs to be “tailed” by an interest rate factor. The object of the tail is to offset the accrual of interest on margin variations, the daily debits and credits implied by the marking to market of futures contracts². There are two impediments to tailing which preclude a perfectly riskless arbitrage. First, The resulting futures position is likely to contain a fraction of a contract and is not tradeable. Second,

²

the optimal tail factor can only be estimated because it is equal to the inverse of 1 plus the uncertain simple rate of interest from the next date to the futures expiration date.

Financing: A program trade entails borrowing or lending the value of the stocks in the equivalent DJIA basket. The fair value equation presumes both can be done at the same rate but borrowing and lending rates differ. The rate is usually a rebate rate a "haircut" below market lending rates. Similarly, the borrowing rate is a haircut above prevailing market borrowing rates. The fair value equality is therefore replaced by two inequalities, the first based on the borrowing rate and the second on the lending rate. The inequalities determine an arbitrage band within which it is not profitable to arbitrage. Since different arbitrageurs don't face the same lending or borrowing rates, each has a different arbitrage band.

Transaction costs: The principal costs of an arbitrage are bid-ask spreads and brokerage fees. The arbitrage is only profitable if these costs are covered. Hence transaction costs have to be added to the arbitrage bounds. Denoting the borrowing and lending rates by r_B and r_L , the fair value equation is replaced by the following arbitrage band. It is trader-specific because traders face different interest rates and brokerage costs.

- Upper bound = $F^u = DJIA (1+r_B T/360) - \text{dividends} +$
futures bid-ask + NYSE bid-ask + total brokerage
- Lower Bound = $F_l = DJIA (1+r_L T/360) - \text{dividends} -$
(futures bid-ask + NYSE bid-ask + total brokerage)

Decimalization has narrowed the NYSE bid-ask spread to a penny and brokerage for bulk transactions to about \$.02. An estimate of the round-turn spread and brokerage cost for the DJIA basket equivalent to one full size futures is approximately $69 \times 30 \times .03 = \62.1 . On the futures side, the average spread is now three ticks, or \$30, and brokerage varies from 0 to \$2. Adding all these, a generous estimate of the total round-turn cost of the arbitrage is close to \$94. This adds approximately 9.5 index points above the fair value based on the borrowing rate and 9.5 index points below the fair value based on the lending rate.

Execution

For NYSE stocks, orders can be routed through SuperDot for execution at the individual specialist posts, or they can be executed automatically via NYSEDirect. NYSEDirect orders are limited to 1,052 shares per stock and execution is not guaranteed. If the arbitrage is based on CBOT mini-sized Dow futures, execution is electronic. For the pit-traded contract, orders can be routed automatically. Multiple routes can be used: the CBOT's TOPS system which can route orders either to the Electronic Clerk in the pit or a broker's TOP terminal at a floor trading desk, wireless transmission to headsets, and wireless transmission to hand-held terminals.

In a perfect arbitrage, the two legs of the position are executed simultaneously at the prices which triggered the trade. They are also unwound at identical times and prices. Immediate and simultaneous execution is facilitated by electronic trading, the special expiration rules for stock index futures and the small number and liquidity of stocks in the DJIA. There is nevertheless room for slippage. Potential sources of slippage include stale quotes that can falsely signal an

arbitrage opportunity, market congestion and circuit breakers³. Arbitrage programs short stocks are also slowed by the uptick rule which restricts shorting stocks after a price decrease. Since congestion and circuit-breakers rise with market volatility, so does slippage.

Concluding Comments

This review has focused on a classical form of stock index futures arbitrage: a two-way arbitrage that pairs simultaneous stock and CBOT DJIA futures transactions and that is triggered by economically significant deviations of the futures price from its fair value. The introduction of fully-electronic CBOT mini-sized DJIA futures has opened new arbitrage opportunities from this type of mispricing and has increased the efficiency of executing the arbitrage program. Variations on this theme abound: a one way arbitrage between the cash market and the equivalent futures position, arbitrage of DJIA futures against DJIA options, Diamonds, (see "4Cs of Diamond Arbitrage" in Advanced Strategies under Education at www.cbot.com/dow) and arbitrage against S&P 500 futures.

Endnotes

¹Stock index arbitrage programs accounts for only a portion of total program trading, which is defined as the purchase or sale of a basket of at least 15 stocks with a total value of \$1 million or more. In the first week of July 2002, program trading represented 32.0 percent of NYSE average daily volume.

²Unlike forward contracts, futures are marked to market. This means gains and losses are settled daily in cash. The payments are called margin variation and accrue until expiration. This implies the cumulative dollar return on a CBOT DJIA futures is not equal to the net price change $(F_T - F_0)$ as for a forward contract, it is equal to $\sum_{t=0}^{T-1} (F_{t+1} - F_t) / (1 + r_{t+1,T})$, where T is the expiration date and $r_{t+1,T}$ is the simple rate of interest from $t+1$ to T . If rates of interest were equal to zero, this would reduce to the return on the forward position, but in general, the futures return includes the interest on margin variations.

³In addition to the trading halts described at www.cbot.com/dow, circuit breakers include NYSE Rule 80A trading collars which restrict index arbitrage orders in component stocks of the S&P 500 (these includes all the DJIA stocks) when the DJIA varies by 2% points or more from its value at the previous close. Specifically, buy orders can be executed only on a minus or zero-plus tick and sell orders on a plus or zero-plus tick. The restriction is in effect until the DJIA moves back to within 1% of the previous closing value of the DJIA.

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